A NOTE ON THE ESTIMATION OF MEAN OF A SYMMETRICAL POPULATION

BY

S.R. SRIVASTAVA*

Department of Mathematics, University of Sulaimaniyah, Iraq

AND

BANARASI

Harish Chandra Post Graduate College, Varanasi

(Received : July, 1980)

SUMMARY

A refinement of Sahai and Ray estimator has been suggested for estimating the mean $\bar{Y}$ of a population with variance σ^2. It has the same mean square error but smaller bias than the one suggested by Sahai and Ray.

INTRODUCTION

For estimating the population mean, $\bar{Y}$ Sahai and Ray [3] proposed the estimator

$$te^* = \bar{y} + \frac{s^2\,\bar{y}}{(n\bar{y}^2 + s^2)} \qquad \ldots(1)$$

and obtained expressions for bias and mean square error to the order $0(n^{-2})$.

$$\text{Bias } (te^*) = \frac{c\bar{Y}}{n} \qquad \ldots(2)$$

and

$$M(te^*) = \frac{\sigma^2}{n}\left(1 - \frac{c}{n}\right) \qquad \ldots(3)$$

$\bar{y}$ is the sample mean, s^2 the sample variance, $c\left(=\frac{\sigma^2}{\bar{Y}^2}\right)$ the square of the coefficient of variation (supposed to be known) and n is the size of the sample. If c is unknown, then it is a common practice to replace c by its sample value $Ce = \frac{s^2}{\bar{y}^2}$.

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* On leave of absence from B.H.U., Varanasi.

Now we consider the estimator

$$te^{**}=\bar{y}+\frac{s^2\bar{y}}{(n\bar{y}^2+s^2)}-\frac{s^4\bar{y}}{(n\bar{y}^2+s^2)^2} \qquad \ldots(4)$$

To evaluate the bias and mean square error of the estimator te^{**} to the order $0(n^{-2})$, let

$$\bar{y}=\bar{Y}+U, \qquad s^2=\sigma^2+V \qquad \ldots(5)$$

Where U and V are order $0(n^{-\frac{1}{2}})$ with $E(U)=E(V)=0$. Then upto the order $0(n^{-2})$, we have

$$te^{**}=\bar{y}+\frac{s^2\,\bar{y}}{(n\bar{y}^2+s^2)}-\frac{s^4\,\bar{y}}{(n\bar{y}^2+s^2)^2}$$

$$=\bar{y}+\left(\frac{s^2}{n\bar{y}}\right)\left(1-\frac{s^2}{n\bar{y}^2}\right)-\frac{s^4\,\bar{y}}{n\bar{y}^2+s^2)^2}$$

$$=(\bar{Y}-U)+\bar{Y}\left\{\left(c+\frac{V}{\bar{Y}^2}\right)\left(1-\frac{U}{\bar{Y}}+\frac{U^2}{\bar{Y}^2}\right)\Big/ n-\frac{c^2}{n^2}\right\}-\frac{c^2\,\bar{Y}}{n^2}$$

$$=\bar{Y}\left(1+\frac{U}{\bar{Y}}\right)+\bar{Y}\left(\frac{c}{n}+\frac{V}{n\bar{Y}}-\frac{cU}{n\bar{Y}}+\frac{cU^2}{n\bar{Y}^2}-\frac{c^2}{n^2}-\frac{UV}{n\bar{Y}^3}+\frac{U^2\,V}{n\bar{Y}^4}\right)-\frac{c^2\bar{Y}}{n^2}$$

So that

$$E(te^{**}-\bar{Y})=\frac{c\bar{Y}}{n}+\frac{c^2\,\bar{Y}}{n^2}-\frac{2c^2\,\bar{Y}}{n^2}$$

$$=\frac{c\bar{Y}}{n}\left(1-\frac{c}{n}\right)$$

$$\therefore \text{ Bias }=\frac{c\bar{Y}}{n}\left(1-\frac{c}{n}\right) \qquad \ldots(6)$$

and

$$M(te^{**})=\frac{\sigma^2}{n}\left(1-\frac{c}{n}\right) \qquad \ldots(7)$$

From equation (2), (3), (6) and (7) it is seen that

$$B(te^{**})<B(te^{*}),$$

and

$$M(te^{**})=M(te^{*})$$

Therefore our estimator te^{**} is superior to the estimator te^{*} in the sense that it has a smaller bias than the estimator [3]. The percentage reduction in the bias by the proposed estimator is given by

$$\frac{B(te^{*})-B(te^{**})}{B(te^{*})}\times 100$$

$$=\frac{\frac{c\bar{Y}}{n}-\frac{c\bar{Y}}{n}\left(1-\frac{c}{n}\right)\times 100}{\frac{c\bar{Y}}{n}}$$

$$=\frac{c}{n}\times 100 \qquad \text{.. (8)}$$

From (8), it is clear that the reduction in bias is large when c is large, meaning thereby that for small sample size the proposed estimator te^{**} is superior to the estimator te^{*}.

REFERENCES

[1] Srivastava, V.K. (1974) : 'On the use of the Coefficient of variation in estimating mean'. *Jour. Ind. Soc. Agri-Stat* **26**, (2), 33-36.

[2] Upadhyaya, L.N. & Srivastava, S.R. (1976) : A note on the use of the coefficient of variation in estimating means. *Jour. Ind. Soc. Agri-Stat-* **26**(2), 97-99.

[3] Sahai, A. and Ray, S.K. (1979) : On estimating the mean of symmetrical population. *Jour. Ind. Soc. Agri-Stat* **31**(3)45-48.